

# Cryptography

## ITS335: IT Security

Sirindhorn International Institute of Technology  
Thammasat University

Prepared by Steven Gordon on 2 January 2015  
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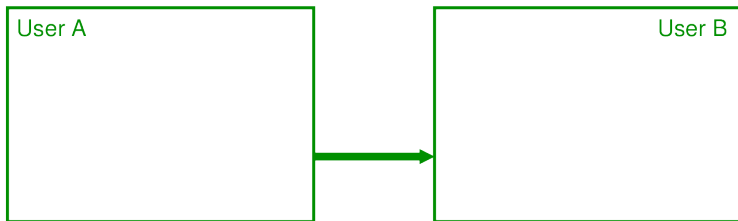


# Encryption for Confidentiality

- ▶ Aim: assure confidential information not made available to unauthorised individuals (data confidentiality)
- ▶ How: encrypt the original data; anyone can see the encrypted data, but only authorised individuals can decrypt to see the original data
- ▶ Used for both sending data across network and storing data on a computer system

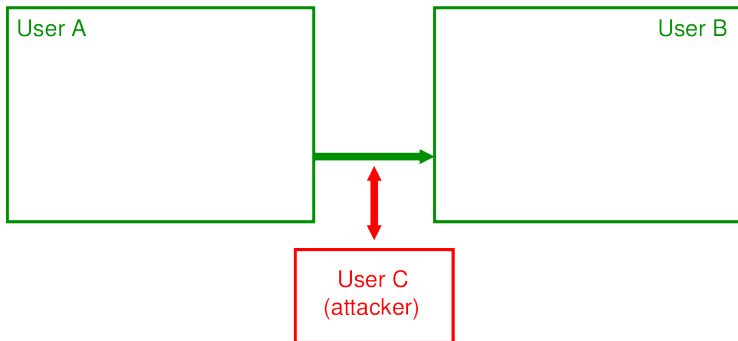


# Model of Encryption for Confidentiality



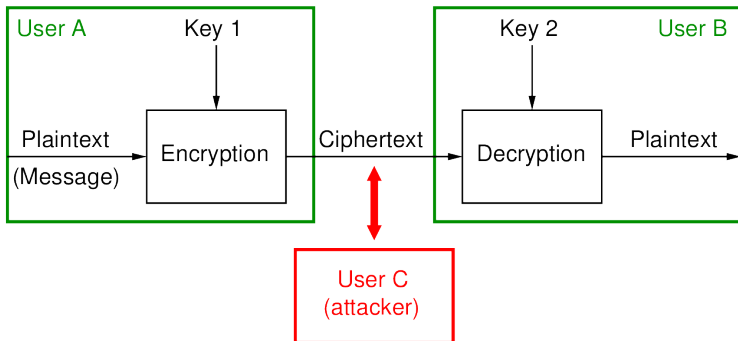


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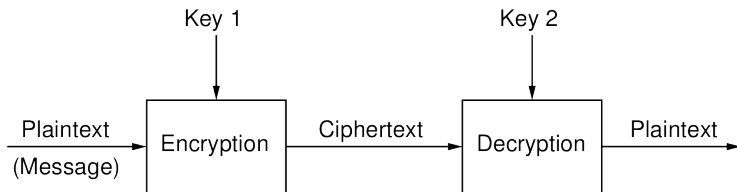


# Model of Encryption for Confidentiality





# Model of Encryption for Confidentiality





# Terminology

**Plaintext** original message

**Ciphertext** encrypted or coded message

**Encryption** convert from plaintext to ciphertext  
(enciphering)

**Decryption** restore the plaintext from ciphertext  
(deciphering)

**Key** information used in cipher known only to  
sender/receiver

**Cipher** a particular algorithm (cryptographic system)

**Cryptography** study of algorithms used for encryption

**Cryptanalysis** study of techniques for decryption without  
knowledge of plaintext

**Cryptology** areas of cryptography and cryptanalysis



# Requirements and Assumptions

Requirements for secure use of symmetric encryption:

1. Strong encryption algorithm: Given the algorithm and ciphertext, an attacker cannot obtain key or plaintext
2. Sender/receiver know secret key (and keep it secret)

Assumptions:

- ▶ Cipher is known
- ▶ Secure channel to distribute keys



# Characterising Cryptographic Systems

## Operations used for encryption:

**Substitution** replace one element in plaintext with another

**Transposition** re-arrange elements

**Product systems** multiple stages of substitutions and transpositions

## Number of keys used:

**Symmetric** sender/receiver use same key (single-key, secret-key, shared-key, conventional)

**Public-key** sender/receiver use different keys (asymmetric)

## Processing of plaintext:

**Block cipher** process one block of elements at a time

**Stream cipher** process input elements continuously



# Example Substitution Cipher: Caesar Cipher

**Encrypt** Shift plaintext letters  $K$  positions to right  
(wrapping where necessary)



# Example Transposition Cipher: Rail-Fence

**Encrypt** Plaintext letters written in diagonals over  $K$  rows; ciphertext obtained by reading row-by-row



# Example Product System

Encrypt Repeat following steps  $n$  times:

1. Apply **Vigenere** cipher with  $K_{n,1}$
2. Apply Rail-fence cipher with  $K_{n,2}$



# Attacks

## Goal of the Attacker

- ▶ Discover the plaintext (good)
- ▶ Discover the key (better)

## Assumed Attacker Knowledge

- ▶ Ciphertext
- ▶ Algorithm
- ▶ Other pairs of (plaintext, ciphertext) using same key

## Attack Methods

**Brute-force attack** Try every possible key on ciphertext

**Cryptanalysis** Exploit characteristics of algorithm to deduce plaintext or key

Assumption: **attacker can recognise correct plaintext**



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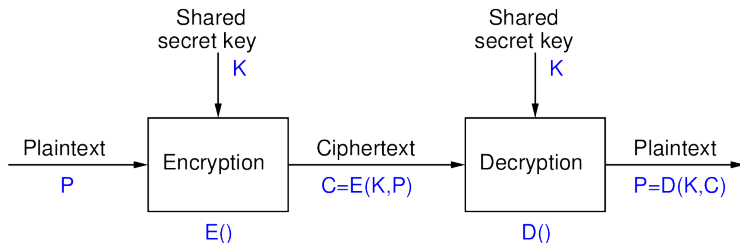
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# Symmetric Key Encryption for Confidentiality



## Requirements

- ▶ Strong encryption algorithm: given algorithm, ciphertext and known pairs of (plaintext, ciphertext), attacker should be unable to find plaintext or key
- ▶ Shared secret keys: sender and receiver both have shared a secret key; no-one else knows the key



# Block vs Stream Ciphers

## Block Ciphers

- ▶ Encrypt block of plaintext at a time, typically 64 or 128 bits
- ▶ Slow algorithms/implementations
- ▶ Can re-use keys

## Stream Ciphers

- ▶ Encrypt 1 byte of plaintext at a time
- ▶ Encryption performed by XOR plaintext with keystream (created by pseudo-random number generator)
- ▶ Fast algorithms/implementations
- ▶ Cannot re-use keys

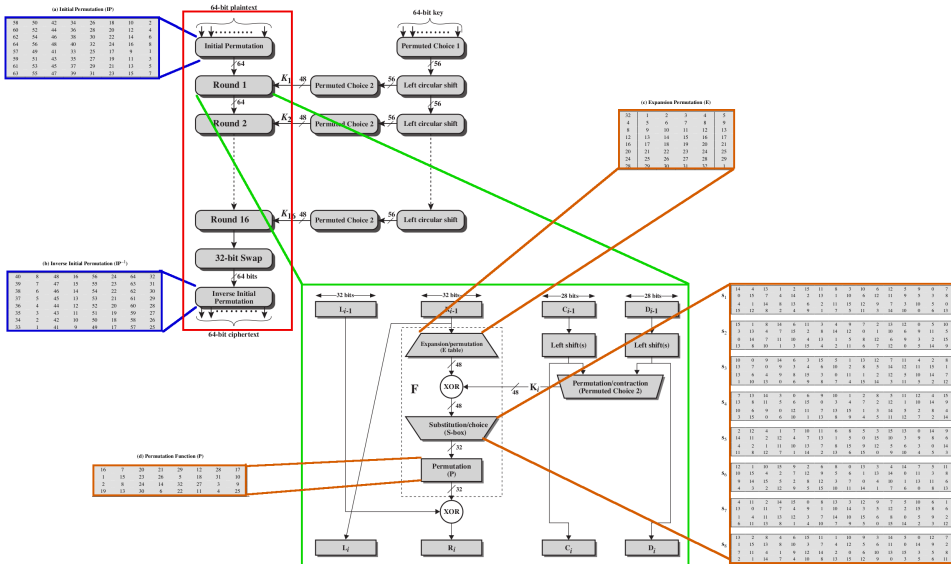


# Data Encryption Standard (DES)

- ▶ Designed by IBM and NSA; standardised by NIST in 1977 as FIPS-46
  - ▶ 1999: NIST recommended Triple-DES; DES only for legacy systems
  - ▶ 2005: FIPS-46 standard withdrawn
- ▶ Block size: 64 bits
- ▶ Key length: 56 bits (64 bits, but 8 are parity)
- ▶ Initial and final permutations, then 16 rounds, each involving permutations and substitutions
- ▶ Feistel structure
- ▶ Decryption is almost identical to encryption → single implementation for both algorithms
- ▶ **Key size is insecure**; algorithm considered secure
- ▶ Status: not recommended



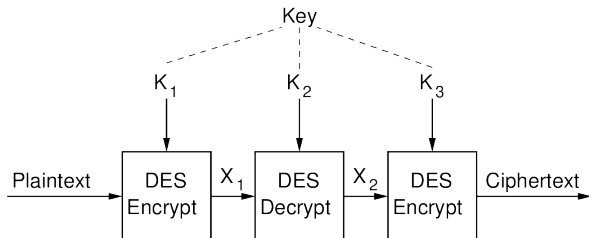
# DES Encryption Operations





# Triple-DES (3DES)

- ▶ Standardised by ANSI/NIST in 1998/99
- ▶ Applies DES three times: Encrypt, Decrypt, Encrypt
- ▶ Block size: 64 bits
- ▶ Key length: 168 bits (options for 112 and 56 bits)
- ▶ **Three times slower than DES**
- ▶ Status: banks still use in many applications; available as an option in many products





# Advanced Encryption Standard (AES)

- ▶ NIST held competition to select algorithm to replace DES/3DES in 1997
  - ▶ Won by Rijndael algorithm by Rijmen and Daemen
  - ▶ 2001: Standardised as FIPS-197
- ▶ Block size: 128
- ▶ Key length: 128, 192, 256 bits
- ▶ Substitution-permutation network
- ▶ Status: used in many products, e.g. WiFi (WPA), full disk encryption (BitLocker, FileVault2, dm-crypt, LUKS), Internet security (HTTPS), ...



## Other Symmetric Encryption Algorithms

- ▶ Blowfish (Schneier, 1993): 64 bit blocks/32–448 bit keys; Feistel structure
- ▶ Twofish (Schneier et al, 1998): 128/128, 192, 256; Feistel structure
- ▶ Serpent (Anderson et al, 1998): 128/128, 192, 256; Substitution-permutation network
- ▶ Camellia (Mitsubishi/NTT, 2000): 128/128, 192, 256; Feistel structure
- ▶ IDEA (Lai and Massey, 1991): 64/128
- ▶ CAST-128 (Adams and Tavares, 1996): 64/40–128; Feistel structure
- ▶ CAST-256 (Adams and Tavares, 1998): 128/up to 256; Feistel structure
- ▶ RC5 (Rivest, 1994): 32, 64 or 128/up to 2040; Feistel-like structure
- ▶ RC6 (Rivest et al, 1998): 128/128, 192, 256; Feistel structure



## Assumptions: Symmetric Key Encryption

- ▶ The same secret key,  $K$ , is used for encryption,  $E()$ , and decryption,  $D()$ . The secret is shared between two entities, i.e.  $K_{AB}$ .
- ▶ Encrypting plaintext,  $P$ , with a key, produces ciphertext  $C$ , e.g.  $C = E(K_{AB}, P)$ .
- ▶ Decrypting ciphertext with the correct key will produce the original plaintext. The decrypter will be able to recognise that the plaintext is correct (and therefore the key is correct). E.g.  $P = D(K_{AB}, C)$ .
- ▶ Decrypting ciphertext using the incorrect key will *not* produce the original plaintext. The decrypter will be able to recognise that the key is wrong, i.e. the decryption will produce unrecognisable output.



# Attacks on Block Ciphers

## Brute Force Attack

- ▶ Approach: try all keys in key space
- ▶ Metric: number of operations (time)
- ▶  $k$  bit key requires  $2^k$  operations
- ▶ Depends on key length and computer speed

## Cryptanalysis

- ▶ Approach: Find weaknesses in algorithms
- ▶ Methods: Linear cryptanalysis, differential cryptanalysis, meet-in-the-middle attack, side-channel attacks ...
- ▶ Metrics:
  - ▶ Number of operations
  - ▶ Amount of memory
  - ▶ Number of known plaintexts/ciphertexts



# Brute Force Attacks on Block Ciphers

Encrypt for  
Confidentiality

Symmetric Key

Authentication

Public Key

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Summary

Key length	Key space	Worst case time at speed:		
		$10^9/\text{sec}$	$10^{12}/\text{sec}$	$10^{15}/\text{sec}$
32	$2^{32}$	4 sec	4 ms	4 us
56	$2^{56}$	833 days	20 hrs	72 sec
64	$2^{64}$	584 yrs	213 days	5 sec
128	$2^{128}$	$10^{22}$ yrs	$10^{19}$ yrs	$10^{16}$ yrs
192	$2^{192}$	$10^{41}$ yrs	$10^{38}$ yrs	$10^{35}$ yrs
256	$2^{256}$	$10^{60}$ yrs	$10^{57}$ yrs	$10^{54}$ yrs
26!	$2^{88}$	$10^{10}$ yrs	$10^7$ yrs	$10^4$ yrs

Age of Earth:  $4 \times 10^9$  years

Age of Universe:  $1.3 \times 10^{10}$  years



# How Fast/Expensive is a Brute Force Attack Today?



# Cryptanalysis on Block Ciphers

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Cipher	Method	Key space	Required resources:		
			Time	Memory	Known data
DES	Brute force	$2^{56}$	$2^{56}$	-	-
3DES	MITM	$2^{168}$	$2^{111}$	$2^{56}$	$2^2$
3DES	Lucks	$2^{168}$	$2^{113}$	$2^{88}$	$2^{32}$
AES 128	Biclique	$2^{128}$	$2^{126.1}$	$2^8$	$2^{88}$
AES 256	Biclique	$2^{256}$	$2^{254.4}$	$2^8$	$2^{40}$

- ▶ Known data: chosen pairs of (plaintext, ciphertext)
- ▶ MITM: Meet-in-the-middle
- ▶ Lucks: S. Lucks, Attacking Triple Encryption, in *Fast Software Encryption*, Springer, 1998
- ▶ Biclique: Bogdanov, Khovratovich and Rechberger, Biclique Cryptanalysis of the Full AES, in *ASIACRYPT2011*, Springer, 2011



# Assumptions: Knowledge of Attacker

- ▶ All algorithms used in cryptography, e.g. encryption/decryption algorithms, hash functions, are public.
- ▶ An attacker knows which algorithm is being used, and any public parameters of the algorithm.
- ▶ An attacker can intercept any message sent across a network.
- ▶ An attacker does not know secret values (e.g. symmetric secret key  $K_{AB}$  or private key  $PR_A$ ).
- ▶ Brute force attacks requiring greater than  $2^{80}$  operations are impossible.

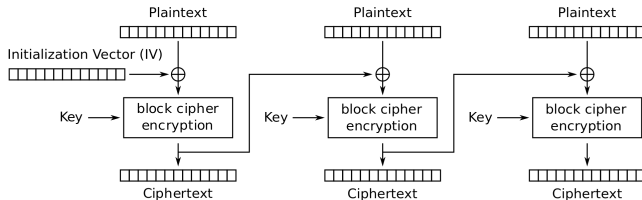


# Using Block Ciphers on Real Data

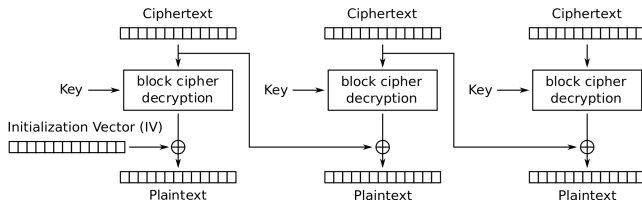
- ▶ Block ciphers typically operate on 64 or 128 bit blocks
- ▶ **Modes of operation** are used to apply ciphers on multiple blocks
  - ▶ Electronic Code Book (ECB), Cipher Block Chaining (CBC), Cipher Feedback Mode (CFB), Output Feedback Mode (OFB), Counter (CTR), XTS-AES
- ▶ Trade-offs: security, parallelism, error propagation
- ▶ Often require Initialisation Vector (IV)
- ▶ CFB, OFB and CTR can turn block cipher into stream cipher



# Mode of Operation Example: CBC



Cipher Block Chaining (CBC) mode encryption



Cipher Block Chaining (CBC) mode decryption



# Mode of Operation Example: CTR

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Symmetric Key

Authentication

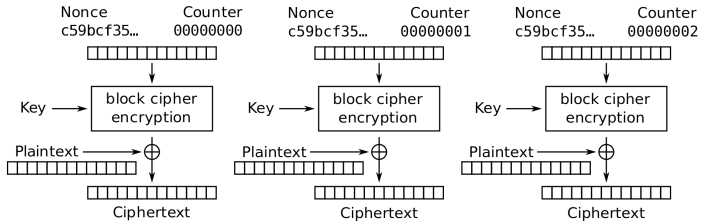
Public Key

Key Management

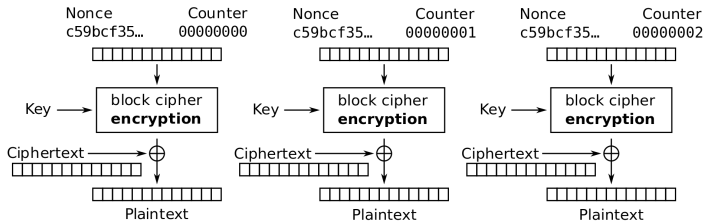
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Counter (CTR) mode encryption

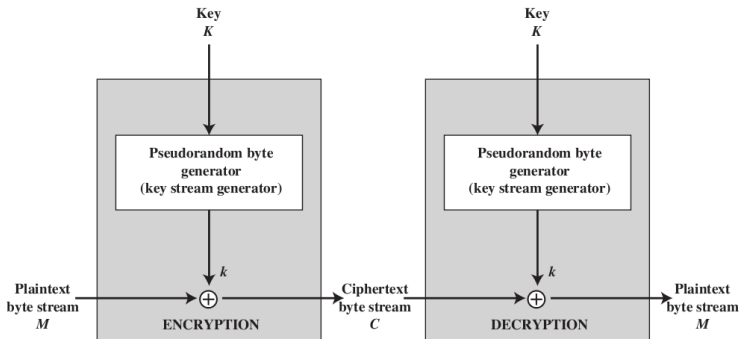


Counter (CTR) mode decryption



# Stream Ciphers

- ▶ Encrypt one byte at a time by XOR with pseudo-random byte (**keystream**)
- ▶ Generally faster implementations than block ciphers
- ▶ Keystream should not repeat (large period); use different key or nonce when re-using cipher





## Example Stream Cipher: RC4

- ▶ Designed by Ron Rivest in 1987
- ▶ Used in secure web browsing and wireless LANs
- ▶ Can use variable size key: 8 to 2048 bits
- ▶ Several theoretical limitations of RC4



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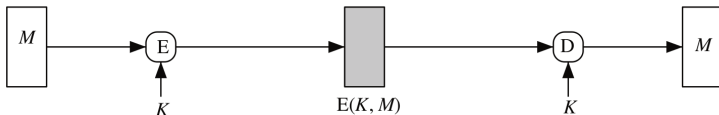
# Authentication

- ▶ Receiver wants to verify:
  1. Contents of the message have not been modified (*data authentication*)
  2. Source of message is who they claim to be (*source authentication*)
- ▶ Different approaches available:
  - ▶ Symmetric Key Encryption
  - ▶ Message Authentication Codes
  - ▶ Hash Functions
  - ▶ Public Key Encryption (see Digital Signatures)



# Authentication using Symmetric Key Encryption

- ▶ Assumption: decryption using wrong key or modified ciphertext will produce unintelligible output
- ▶ Symmetric key encryption can provide: data authentication and source authentication (as well as confidentiality)



Credit: Figure 12.1(a) in Stallings, *Cryptography and Network Security*, 5th Ed., Pearson 2011

- ▶ However, typically authentication is performed separately to encryption for confidentiality
  - ▶ Avoid overhead of using encryption when not needed



# Authentication using Message Authentication Codes

- ▶ Append small, fixed-size block of data to message: cryptographic checksum or MAC

$$\text{MAC} = F(K, M)$$

$M$  = input message

$F$  = MAC function

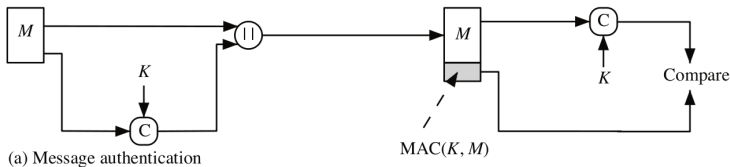
$K$  = shared secret key of  $k$  bits

$\text{MAC}$  = message authentication code (or tag) of  $n$  bits

- ▶ MAC function also called *keyed hash function*
- ▶ MAC function similar to encryption, but does not need to be reversible
  - ▶ Easier to design stronger MAC functions than encryption functions



# Authentication using Message Authentication Codes



Credit: Figure 12.4(a) in Stallings, *Cryptography and Network Security*, 5th Ed., Pearson 2011



# MAC Algorithms

- ▶ Data Authentication Algorithm (DAA): based on DES; considered insecure
- ▶ Cipher-Based Message Authentication Code (CMAC): mode of operation used with Triple-DES and AES
- ▶ OMAC, PMAC, UMAC, VMAC, ...
- ▶ HMAC: MAC function derived from cryptographic hash functions
  - ▶ MD5/SHA are fast in software (compared to block ciphers)
  - ▶ Libraries for hash functions widely available
  - ▶ Security of HMAC depends on security of hash function used



# MAC Attacks

## Security Requirement

- ▶ Key is secret and difficult to find from pairs of (M, MAC)
- ▶ Given pairs of (M, MAC), difficult to find the MAC of another message

## Brute Force Attacks on MACs

- ▶ Option 1: Try all possible keys for one or more pairs of (MAC, M); effort  $\approx 2^k$
- ▶ Option 2: Try many values of M to find correct MAC; effort  $\approx 2^n$
- ▶ Effort to break MAC:  $\min(2^k, 2^n)$



## Assumptions: Authentication with Symmetric Key and MACs

- ▶ An entity receiving ciphertext that successfully decrypts with symmetric secret key  $K_{AB}$  knows that the original message has not been modified and that it originated at one of the owners of the secret key (i.e.  $A$  or  $B$ ).
- ▶ An entity receiving a message with attached MAC that successfully verifies, knows that the message has not been modified and originated at one of the owners of the MAC secret key.

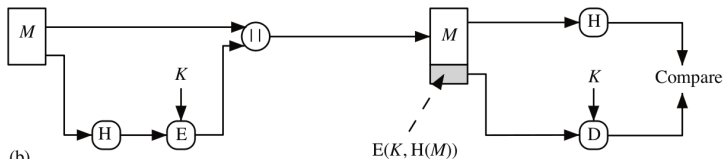


# Authentication using Hash Functions

- ▶ **Hash function**  $H$ : variable-length block of data  $M$  input; fixed-size hash value  $h = H(M)$  output
- ▶ Applying  $H$  to large set of inputs should produce evenly distributed and random looking outputs
- ▶ **Cryptographic hash function**: computationally infeasible to find:
  1.  $M$  that maps to known  $h$  (one-way property)
  2.  $M_1$  and  $M_2$  that produce same  $h$  (collision-free property)
- ▶ Append hash value to message; receiver verifies if message changed



# Example of Authentication with Hash functions



Credit: Figure 11.2(b) in Stallings, *Cryptography and Network Security*, 5th Ed., Pearson 2011



# Hash Algorithms: MD5

- ▶ Message Digest algorithm 5, developed by Ron Rivest in 1991
- ▶ Standardised by IETF in RFC 1321
- ▶ Generates 128-bit hash
- ▶ Was commonly used by applications, passwords, file integrity; **no longer recommended**
- ▶ Collision and other attacks possible; tools publicly available to attack MD5



# Hash Algorithms: SHA

- ▶ Secure Hash Algorithm, developed by NIST
- ▶ Standardised by NIST in FIPS 180 in 1993
- ▶ Improvements over time: SHA-0, SHA-1, SHA-2, SHA-3
- ▶ SHA-1 (and SHA-0) are considered insecure; **no longer recommended**
- ▶ SHA-2 considered secure
- ▶ SHA-3 in begin standardised by NIST

	SHA-1	SHA-224	SHA-256	SHA-384	SHA-512
<b>Message Digest Size</b>	160	224	256	384	512
<b>Message Size</b>	$< 2^{64}$	$< 2^{64}$	$< 2^{64}$	$< 2^{128}$	$< 2^{128}$
<b>Block Size</b>	512	512	512	1024	1024
<b>Word Size</b>	32	32	32	64	64
<b>Number of Steps</b>	80	64	64	80	80



# Hash Attacks

## Security Requirement

**Preimage resistant:** For any given  $h$ , computationally infeasible to find  $y$  such that  $H(y) = h$   
(*one-way property*)

**Second preimage resistant:** For any given  $x$ , computationally infeasible to find  $y \neq x$  with  $H(y) = H(x)$   
(*weak collision resistant*)

**Collision resistant:** Computationally infeasible to find any pair  $(x, y)$  such that  $H(x) = H(y)$   
(*strong collision resistant*)

## Brute Force Attacks

- ▶ Depend on hash value length of  $n$  bits
- ▶ Preimage and second preimage resistant:  $2^n$
- ▶ Collision resistant:  $2^{n/2}$



# Required Properties when using Hash Functions

Not all applications of hash functions require all properties

	Preimage Resistant	Second Preimage Resistant	Collision Resistant
Hash + digital signature	yes	yes	yes*
Intrusion detection and virus detection		yes	
Hash + symmetric encryption			
One-way password file	yes		
MAC	yes	yes	yes*

\* Resistance required if attacker is able to mount a chosen message attack

Credit: Table 11.2 in Stallings, *Cryptography and Network Security*, 5th Ed., Pearson 2011



# How Fast/Expensive is a MD5 Collision Attack Today?



# Assumptions: Hash Functions

- ▶ A cryptographic hash function,  $H()$ , takes a variable sized input message,  $M$ , and produces a fixed size, small output hash,  $h$ , i.e.  $h = H(M)$ .
- ▶ Given a hash value,  $h$ , it is impossible to find the original message  $M$ .
- ▶ Given a hash value,  $h$ , it is impossible to find another message  $M'$  that also has a hash value of  $h$ .
- ▶ It is impossible to find two messages,  $M$  and  $M'$ , that have the same hash value.



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# Birth of Public-Key Cryptosystems

- ▶ Beginning to 1960's: permutations and substitutions (Caesar, rotor machines, DES, ...)
- ▶ 1960's: NSA secretly discovered public-key cryptography
- ▶ 1970: first known (secret) report on public-key cryptography by CESG, UK
- ▶ 1976: Diffie and Hellman public introduction to public-key cryptography
  - ▶ Avoid reliance on third-parties for key distribution
  - ▶ Allow digital signatures
- ▶ 1978: Rivest, Shamir and Adleman created RSA



# Principles of Public-Key Cryptosystems

- ▶ Symmetric algorithms used same secret key for encryption and decryption
- ▶ Asymmetric algorithms in public-key cryptography use one key for encryption and different but related key for decryption
- ▶ Characteristics of asymmetric algorithms:
  - ▶ Require: Computationally infeasible to determine decryption key given only algorithm and encryption key
  - ▶ Optional: Either of two related keys can be used for encryption, with other used for decryption



# Public and Private Keys

## Public Key

- ▶ For secrecy: used in encryption
- ▶ For authentication: used in decryption
- ▶ Available to anyone

## Private Key

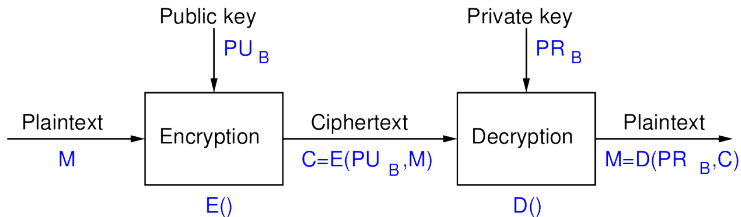
- ▶ For secrecy: used in decryption
- ▶ For authentication: used in decryption
- ▶ Secret, known only by owner

## Public-Private Key Pair

- ▶ User  $A$  has pair of related keys, public and private:  
 $(PU_A, PR_A)$



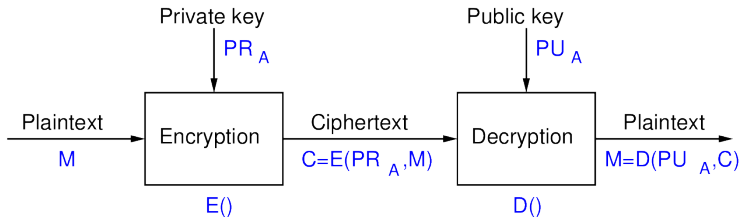
# Confidentiality with Public Key Crypto



- ▶ Encrypt using receiver's public key
- ▶ Decrypt using receiver's private key
- ▶ Only the person with private key can successfully decrypt



# Authentication with Public Key Crypto



- ▶ Encrypt using senders private key
- ▶ Decrypt using senders public key
- ▶ Only the person with private key could have encrypted



# Applications of Public Key Cryptosystems

- ▶ Secrecy, encryption/decryption of messages
- ▶ Digital signature, *sign* message with private key
- ▶ Key exchange, share secret session keys

Algorithm	Encryption/Decryption	Digital Signature	Key Exchange
RSA	Yes	Yes	Yes
Elliptic Curve	Yes	Yes	Yes
Diffie-Hellman	No	No	Yes
DSS	No	Yes	No

Credit: Table 9.3 in Stallings, *Cryptography and Network Security*, 5th Ed., Pearson 2011



# Requirements of Public-Key Cryptography

1. Computationally easy for  $B$  to generate pair  $(PU_b, PR_b)$
2. Computationally easy for  $A$ , knowing  $PU_b$  and message  $M$ , to generate ciphertext:

$$C = E(PU_b, M)$$

3. Computationally easy for  $B$  to decrypt ciphertext using  $PR_b$ :

$$M = D(PR_b, C) = D[PR_b, E(PU_b, M)]$$

4. Computationally infeasible for attacker, knowing  $PU_b$  and  $C$ , to determine  $PR_b$
5. Computationally infeasible for attacker, knowing  $PU_b$  and  $C$ , to determine  $M$
6. (Optional) Two keys can be applied in either order:

$$M = D[PU_b, E(PR_b, M)] = D[PR_b, E(PU_b, M)]$$



# Requirements of Public-Key Cryptography

6 requirements lead to need for **trap-door one-way function**

- ▶ Every function value has unique inverse
- ▶ Calculation of function is easy
- ▶ Calculation of inverse is infeasible, unless certain information is known

$$Y = f_k(X) \quad \text{easy, if } k \text{ and } Y \text{ are known}$$

$$X = f_k^{-1}(Y) \quad \text{easy, if } k \text{ and } Y \text{ are known}$$

$$X = f_k^{-1}(Y) \quad \text{infeasible, if } Y \text{ is known but } k \text{ is not}$$

- ▶ What is easy? What is infeasible?
  - ▶ Computational complexity of algorithm gives an indication
  - ▶ Easy if can be solved in polynomial time as function of input



# Public-Key Cryptanalysis

## Brute Force Attacks

- ▶ Use large key to avoid brute force attacks
- ▶ Public key algorithms less efficient with larger keys
- ▶ Public-key cryptography mainly used for key management and signatures

## Compute Private Key from Public Key

- ▶ No known feasible methods using standard computing

## Probable-Message Attack

- ▶ Encrypt all possible  $M'$  using  $PU_b$ —for the  $C'$  that matches  $C$ , attacker knows  $M$
- ▶ Only feasible if  $M$  is short
- ▶ Solution for short messages: append random bits to make it longer



# Example Public-Key Algorithm: RSA

## Key Generation

1. Choose primes  $p$  and  $q$ , and calculate  $n = pq$
2. Select  $e$ :  $\gcd(\phi(n), e) = 1, 1 < e < \phi(n)$
3. Find  $d \equiv e^{-1} \pmod{\phi(n)}$

$n$  and  $e$  are public;  $p$ ,  $q$  and  $d$  are private

## Encryption

Encryption of plaintext  $M$ , where  $M < n$ :

$$C = M^e \bmod n$$

## Decryption

Decryption of ciphertext  $C$ :

$$M = C^d \bmod n$$



# Example Public-Key Algorithm: RSA

- ▶ Created by Ron Rivest, Adi Shamir and Len Adleman in 1978
- ▶ Security of RSA
  1. Brute force attack on  $d$
  2. Factor  $n$  into its two prime factors
  3. Determine  $\phi(n)$  directly, without determining  $p$  or  $q$
  4. Determine  $d$  directly, without determining  $\phi(n)$
- ▶ Factoring is considered the easiest. Some records by length of  $n$ :
  - ▶ 1991: 330 bits (100 digits)
  - ▶ 2003: 576 bits (174 digits)
  - ▶ 2005: 640 bits (193 digits)
  - ▶ 2009: 768 bit (232 digits),  $10^{20}$  operations, 2000 years on single core 2.2 GHz computer
- ▶ Typical length of  $n$ : 1024 bits, 2048 bits, 4096 bits



# Performance of Public Key Cryptography

- ▶ Public key crypto algorithms typically much slower than symmetric key algorithms



## Assumptions: Public Key Encryption

- ▶ There is a pair of keys, public ( $PU$ ) and private ( $PR$ ). One key from the pair is used for encryption, the other is used for decryption. Each entity has their own pair, e.g.  $(PU_A, PR_A)$ .
- ▶ Encrypting a plaintext message,  $M$ , with a key, produces ciphertext  $C$ , e.g.  $C = E(PU_A, M)$ .
- ▶ Decrypting ciphertext with the correct key will produce the original plaintext. The decrypter will be able to recognise that the plaintext is correct (and therefore the key is correct). E.g.  $M = D(PR_A, C)$ .
- ▶ Decrypting ciphertext using the incorrect key will *not* produce the original plaintext. The decrypter will be able to recognise that the key is wrong, i.e. the decryption will produce unrecognisable output.



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# Key Management

## Challenges

- ▶ How to share a secret key?
- ▶ How to obtain someone else's public key?
- ▶ When to change keys?

## Assumptions and Principles

- ▶ Many users wish to communicate securely across network
- ▶ Attacker can intercept any location in network
- ▶ Manual interactions between users are undesirable (e.g. physical exchange of keys)
- ▶ More times a key is used, greater chance for attacker to discover the key



# Exchanging Secret Keys

## Option 1: Manual Exchange of All Keys

- ▶ All users exchange secret keys with all other users manually (e.g. face-to-face)
- ▶ Inconvenient

## Option 2: Manual Exchange of Master Keys

- ▶ All users exchange **master** key with trusted, central entity (e.g. Key Distribution Centre)
- ▶ **Session** keys automatically exchanged between users via KDC
- ▶ Security and performance bottleneck at KDC



# Exchanging Secret Keys

## Option 3: Public Key Cryptography to Exchange Secrets

- ▶ Use public-key cryptography to securely and automatically exchange secret keys
- ▶ Example 1: user A encrypts secret with user B's public key; sends to B
- ▶ Example 2: Diffie-Hellman secret key exchange



# Distributing Public Keys

- ▶ By design, public keys are made public
- ▶ Issue: how to ensure public key of A actually belongs to A (and not someone pretending to be A)
- ▶ Approaches for public key distribution
  1. Public announcement (web page, email, newspaper)
  2. Publish in electronic directory (which manually authenticates users)
  3. Public key authority:
    - ▶ Users manually publish key at authority, and gain authorities public key
    - ▶ Users automatically request other users public keys from authority
  4. Public key certificates
    - ▶ Users manually register with authority
    - ▶ Authority issues certificates to users: users public key signed by authority
    - ▶ Users automatically exchange certificates



# Key Hierarchy and Lifetimes

- ▶ Master keys used to securely exchange session keys
- ▶ Session keys used to securely exchange data
- ▶ Change session keys automatically and regularly
- ▶ Change master keys manually and seldom
- ▶ Session key lifetime:
  - ▶ Shorter lifetime is more secure; but increases overhead of exchanges
  - ▶ Connection-oriented protocols (e.g. TCP): new session key for each connection
  - ▶ Connection-less protocols (e.g. UDP/IP): change after fixed period or certain number of packets sent



# Assumptions: Key Management

- ▶ A secret key can be exchanged between two entities without other entities learning its value.
- ▶ Any entity can obtain the correct public key of any other entity.



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# Digital Signatures

- ▶ Aim of a signature: prove to anyone that a message originated at (or is approved by) a particular user
- ▶ Symmetric key cryptography
  - ▶ Two users,  $A$  and  $B$ , share a secret key  $K$
  - ▶ Receiver of message (user  $A$ ) can verify that message came from the other user ( $B$ )
  - ▶ User  $C$  *cannot* prove that the message came from  $B$  (it may also have come from  $A$ )
- ▶ Public key cryptography can provide signature: only one user has the private key



# Digital Signature Operations (Concept)

## Signing

- ▶ User signs a message by encrypting with own private key

$$S = E(PR_A, M)$$

- ▶ User attaches signature to message

## Verification

- ▶ User verifies a message by decrypting signature with signer's public key

$$M' = D(PU_A, S)$$

- ▶ User then compares received message  $M$  with decrypted  $M'$ ; if identical, signature is verified



# Digital Signature Operations (Practice)

No need to encrypt entire message; encrypt hash of message

## Signing

- ▶ User signs a message by encrypting **hash of message** with own private key

$$S = E(PR_A, H(M))$$

- ▶ User attaches signature to message

## Verification

- ▶ User verifies a message by decrypting signature with signer's public key

$$h = D(PU_A, S)$$

- ▶ User then compares **hash of** received message,  $H(M)$ , with decrypted  $h$ ; if identical, signature is verified



# Digital Signature Algorithms

- ▶ RSA
- ▶ Digital Signature Algorithm (DSA): FIPS-186
- ▶ ECDSA: DSA with elliptic curve cryptography
- ▶ ElGamal signature scheme: DSA is enhancement of ElGamal
- ▶ Bilinear pairing based signatures, e.g. BLS
- ▶ Different hash algorithms can be used; e.g. SHA2
  - ▶ Preimage resistant, second preimage resistant, collision resistant



## Assumptions: Digital Signatures

- ▶ A digital signature of a message  $M$  is the hash of that message encrypted with the signers private key, i.e.  
$$S = E(PR, H(M))$$
- ▶ An entity receiving a message with an attached digital signature knows that that message originated by the signer of the message.



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# Random Numbers

## Examples of Random Numbers in Cryptography

- ▶ Generate keys in public key algorithms
- ▶ Keystream in stream ciphers
- ▶ Generate session keys for symmetric ciphers
- ▶ Authentication and key distribution protocols to prevent replays

## Requirements of Sequence of Random Numbers

- ▶ Randomness, e.g. selecting large prime numbers for RSA involves selecting random numbers and checking that they are not composite
  - ▶ Uniform distribution
  - ▶ Independence
- ▶ Unpredictability, e.g. protocols rely on unpredictable values so attacker cannot generate fake/replay messages



# Random vs Pseudo-random

## Pseudo-random Number Generators (PRNG)

- ▶ Algorithms used to generate random numbers
- ▶ Algorithms are deterministic, therefore numbers produced are not statistically unpredictable or independent
- ▶ However good algorithms produce sequences of number that pass many randomness tests

## True Random Number Generators

- ▶ Use non-deterministic source to produce randomness
  - ▶ Measure ionising radiation events, leaky capacitors, thermal noise from resistor, disk reads of hard disks, ...
- ▶ Require hardware for measurements



# Pseudo Random Number Generators

## Characteristics

- ▶ Seed: initial state of algorithm
- ▶ Period: length of sequence produced before repeating

## Examples

- ▶ Linear congruential generators
- ▶ Linear feedback shift registers
- ▶ Blum Blum Shub
- ▶ Mersenne Twister
- ▶ Stream and block ciphers



# Assumptions: Random Numbers

- ▶ Pseudo-random number generators (PRNG) can generate effectively true random numbers.



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# Key Points

- ▶ Symmetric key encryption used for file and network data confidentiality (AES, 3DES)
- ▶ Public key encryption used for key exchange and source authentication (RSA, ECC, DH, certificates)
- ▶ MAC functions used for data and source authentication (HMAC)
- ▶ Hash function used for data authentication (MD5, SHA)
- ▶ Public key crypto combined with hash functions for digital signatures
- ▶ Random numbers used in many security algorithms and protocols



## Common Principles used in Security

- ▶ *Experience*: Algorithms that have been used over a long period are less likely to have security flaws than newer algorithms.
- ▶ *Performance*: Symmetric key algorithms are significantly faster than public key algorithms.
- ▶ *Performance*: The time to complete a cryptographic operation is linearly proportional with the input data size.
- ▶ *Key Distribution*: Keys should be distributed using automatic means.
- ▶ *Key Re-use*: The more times a key is used, the greater the chance of an attacker discovering that key.
- ▶ *Multi-layer Security*: Using multiple overlapping security mechanisms can increase the security of a system.



# Security Issues

- ▶ Key management and distribution: difficult to confirm that public key belongs to claimed entity
- ▶ Implementation: flaws in software implementations can weaken otherwise secure algorithms
- ▶ Algorithm design: difficult to prove security of algorithms; where the design decisions well motivated, public?



# Areas To Explore

- ▶ Elliptic Curve Cryptography
- ▶ Steganography
- ▶ Quantum Cryptography